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Proceedings of the American Society of Civil Engineers

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JOURNAL

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

A DIRECT METHOD FOR MODEL ANALYSIS

Norbert E. Landdeck[1]
(Proc. Paper 869)

SYNOPSIS

A structural model cut from a single flat sheet of plastic, and SR-4 resistance-type strain gages are combined to offer a rapid solution of a statically indeterminate frame problem. The method of experimental stress analysis, which is described, is considered practical for structural design problems involving short heavy members, frame members where there are sudden changes in the depth of beams, or for the design of highly indeterminate structures.

INTRODUCTION

There are many advantages to be obtained from an experimental stress analysis. A structural model leads to a better understanding of complex structural action. The model automatically considers many refinements ordinarily neglected in the usual theoretical calculations. The effect of thrust and shear deformations, variable moment of inertia, brackets, and the peculiar distribution of stress around corners are all taken into account automatically by the model. Higher mathematics is completely avoided.

The method of model analysis which follows is simple. The mechanical nature of the analysis makes possible a solution by those who have had limited theoretical training in indeterminate structures. Figure 1 shows a simple indeterminate frame which is to be analyzed for extreme fibre stress or strain at point d. A two-dimensional model of the frame is cut from a sheet of plastic. A resistance-type strain gage is cemented to the plastic model at point d. The model is loaded with a single unit load applied at successive points along the frame member. For each point at which the load is applied there is a corresponding strain at d. The measured strains at the strain gage locate points required for drawing the influence line. This is shown on Figures 2 & 3. The strain gage readings are plotted as vertical ordinates with

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the distance between load points as horizontal ordinates. The model need only be loaded at enough points to define the shape of the influence line.

The Influence Line

An influence line may be defined as a line that represents the variation of strain, or some other function,[2] when a single load of unit value moves across the structure. The area between the influence line and the base line or zero line represents the value of the function for continuous uniform loading of unit weight upon the entire frame member. For other loadings, including nonuniform loading, the vertical ordinates are multiplied by the load per unit length on each section of the structure. The area under this new line represents the value of the function for the system of loading under consideration. The area of Figure 4 represents the total strain at point d for the loading shown on

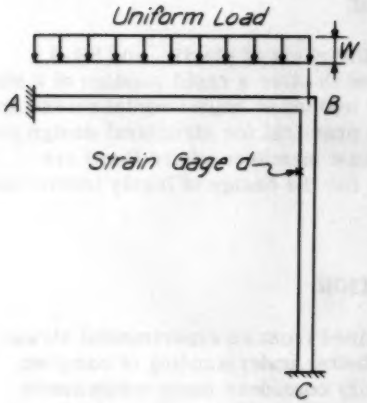


Figure 1

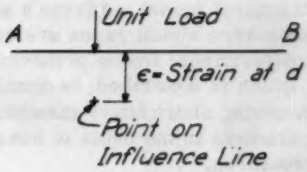


Figure 2

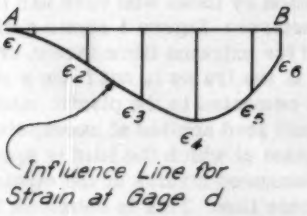


Figure 3

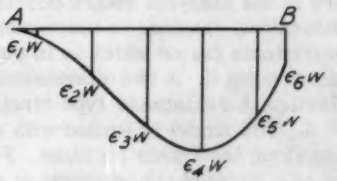


Figure 4

2. Reactions, moments, thrusts, shears, and stresses are frequently represented by influence lines.

Figure 1. In a similar manner, the area of Figure 6 represents the total strain at point d for the load shown on Figure 5. Influence lines have the distinct advantage that they illustrate the effect of loads and their location on the structure, thus leading to a better understanding of the design problems involved. Another advantage is that the structure may be examined for any type of loading or any combination of loadings without further experimental readings after the influence lines have once been established.

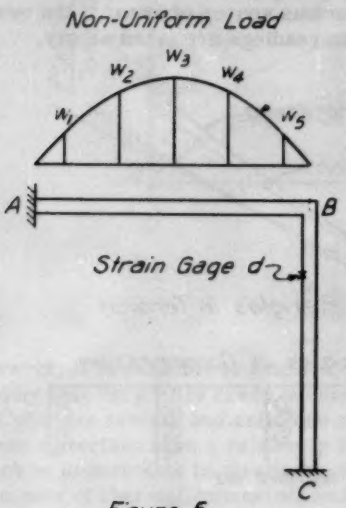


Figure 5

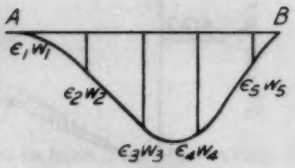


Figure 6

Selection of Plastic for the Model

Plastic is used in the construction of the model because of the ease with which it can be fabricated. The material may be sawed or machined by methods similar to those used on wood and soft metals. Two or more pieces of plastic may be welded together by coating the contact surfaces with solvent-type cement. Another advantage of plastic is that it has a low modulus of elasticity with the result that structural frames of this material will yield large deformations at low loads. Unfortunately, plastic also possesses some undesirable characteristics. The coefficient of expansion is fairly high, with the result that models constructed of this material are sensitive to currents of air and to direct sunlight. Creep is another troublesome characteristic of plastic models. An initial elastic deformation occurs as the model is loaded. If the load is allowed to remain in place, the deformation will continue due to plastic flow.

Methyl methacrylate plastic has been found satisfactory for models and is readily available under the trade name of Plexiglas. Polystyrene is believed to have superior elastic properties but is not readily available in clear sheets of large size. Samples of both plastics were fabricated into tension and compression specimens and were tested to determine if SR-4 strain gages mounted on plastic would give readings proportional to the applied load. The stress-strain relationships for both plastics were found to be essentially linear, as

shown on Figure 7. The modulus of elasticity in tension and the modulus of elasticity in compression may be assumed to be equal without serious error. It has been noted that there may be large differences in the elastic properties of two or more sheets of plastic of the same type, and for this reason it is desirable to construct the model from a single sheet. The modulus of elasticity of the two plastics tested changed with temperature as shown on Figure 8.

The creep or plastic flow for methyl methacrylate and polystyrene is shown on Figure 9. The creep may be a serious source of error if the model is loaded for long intervals and if the strain readings are taken slowly.

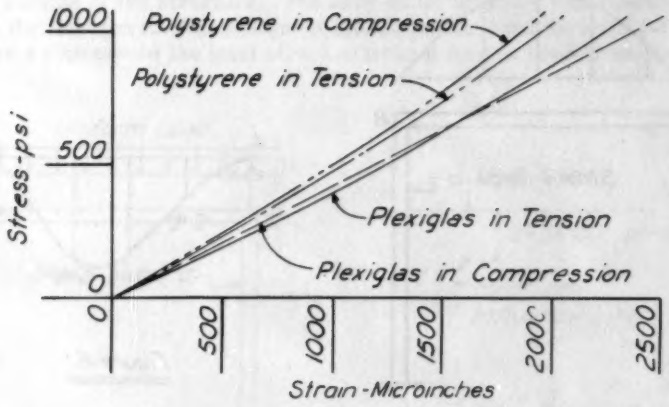


Figure 7

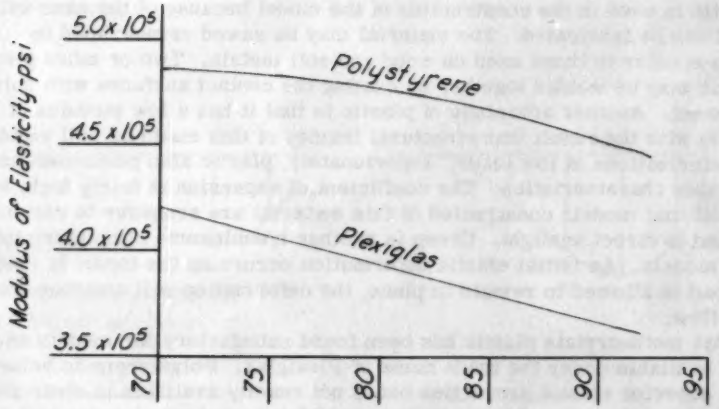


Figure 8

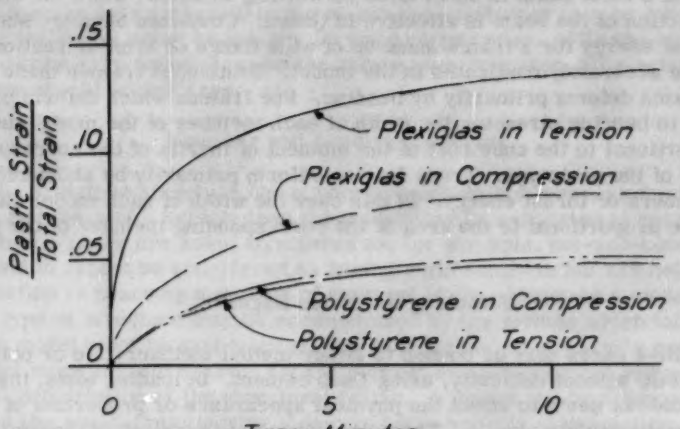


Figure 9

However, it is possible to load the model and to take a strain reading so rapidly that very little creep occurs.

Cellulose acetate and cellulose nitrate are frequently used for models. These materials have a relatively ductile behavior at room temperature which is undesirable in elastic studies. They also have relatively high coefficients of thermal expansion, and the modulus of elasticity changes rapidly with temperature. While not generally recognized, many materials have elastic properties which are not reversible. If the stress-strain line is curved, the stiffness of the material in tension and in compression will not be the same. Cellulose acetate and cellulose nitrate are reported[1] as having a higher modulus in tension than in compression. The reverse is true for some of the other plastics.

Dimensions of the Model

The model is ordinarily two dimensional, for the third dimension represented by the thickness of the plastic sheet is constant. However, the third dimension can be varied by cementing added thickness of plastic unto the main sheet. For satisfactory installation of the SR-4 strain gages, the thickness should not be less than one-eighth inch. The size of the model should be such that no frame member has a width less than one-half inch. Careful workmanship is required with members only one-half inch in width and it generally is desirable to construct the model to a larger scale. As the scale of the model is increased, it becomes less difficult to maintain scale accuracy and to reproduce all details of the prototype. Similitude between the model and the prototype will occur if geometric similarity is maintained. Where the frame members are rectangular sections, as customarily are used in construction with concrete, geometric similarity may be obtained without trouble.

Where the structural members consist of irregular sections, such as rolled steel beams, complete geometric similarity is difficult to obtain. Only

the web of a steel beam is considered as working in shear, while the entire cross section of the beam is effective in thrust. Combined bending, shear and thrust energy for a frame made up of wide flange or similar sections cannot be accurately duplicated in the model. Continuous frames made up of steel beams deform primarily by bending. For frames which deform primarily due to bending stresses, the width of each member of the model should be proportional to the cube root of the moment of inertia of the corresponding member of the prototype. Truss frames deform primarily by shortening of the members or thrust energy. In this case the width of each model member should be proportional to the area of the corresponding member of the prototype.

Attaching the Strain Gages

The SR-4 gages may be bonded to either methyl methacrylate or polystyrene plastic without difficulty, using Duco cement. In loading tests, the Duco cement did not seem to affect the physical appearance or properties of the methyl methacrylate plastic. The polystyrene became slightly clouded in appearance where it came in contact with the cement, but its elastic properties did not seem to change. If one-eighth inch thick, or heavier, plastic is used for the model, SR-4 strain gages, type A-7, may be trimmed to minimum size and bonded to the edge of the plastic. Placing the gage on the edge as shown on Figure 10 proved to be better than placing the gage on the flat surface near the edge. Gages placed on the flat surface tend to give erratic readings because of bending normal to the plane of the model. Also, it is convenient to find the extreme fibre stress. It is logical that the gage should be mounted on the extreme fibre. A thin layer of Duco cement is placed on the back of the gage and on the edge of the model. The gage is then placed in the proper position and held in place for about ten minutes. The gage is allowed to dry overnight before any readings are taken.

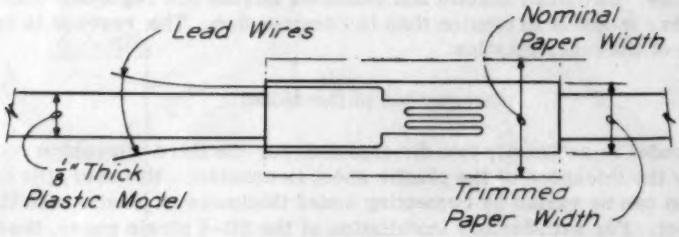


Figure 10

The SR-4 gages may be damaged if the connecting wires to the SR-4 indicator pull on the leads from the gages. Also, any pull which does occur on the gage will also act as a variable and unknown loading of the model. To minimize these tendencies, a radio-type terminal strip may be rigidly mounted on any convenient fixed support close to the gage. Connection between the gage and the terminal strip may be made with No. 28 enameled copper wire. A few zigzag bends are made in the wire to permit deformation with very little force. While the enamel insulation is surprisingly effective, it is best

to arrange the wires so that they do not touch any other conductor. Connections from the terminal strip to the strain gage indicator may be made with plastic insulated, color coded, No. 22 solid copper wire. With the exception of the screw-type connections on the strain gage indicator, all electrical connections should be soldered.

Mounting and Loading the Model

If the prototype structure has supports which do not move, as the abutments and piers of a bridge, then the model may be supported at corresponding points. There are some structures as, for example, cut-and-cover conduits which cannot be considered as having rigid supports but are held in equilibrium in space by a system of external loads. Mounting a model of the latter type of structure may be accomplished by the method which follows.

The model must be supported so that when loaded at any single point, it does not move off into space. Yet the support system must not prevent any normal deformation of the structural frame. A single pin-type support will prevent the model from moving sideways but it will not stop rotation. A second pin-type support is apt to prevent normal deformation of the frame since the distance between the two pins then is fixed. A pin support and a hinge support as shown on Figure 11 will prevent sideways movement, rotation about the pin, and yet will permit normal deformation.

The location of the pin support and the hinge support is not important. The effect of the supports cancels out and becomes zero when external loads are applied to the structure such that the structure is in equilibrium in space. Figure 12 shows a model with external loads acting in a single direction. The model can be loaded with forces acting in a single direction because the loads will be balanced by the reactions R_1 and R_2 at the pin and hinge supports. However, the actual structure does not have the pin and hinge support, so it must have loads acting in the opposite direction, as in Figure 13, and of such

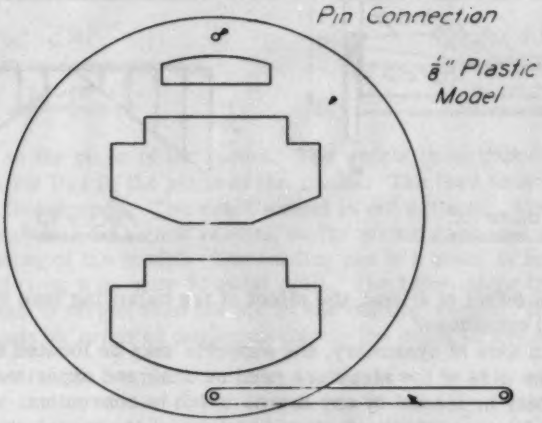


Figure 11

magnitude and distribution as is required to keep the structure in equilibrium. If Figure 13 is superimposed upon Figure 12, the external loads will balance, R_1 will cancel R_3 and R_2 will cancel R_4.

The area between the influence line and the base line determines the value of the function for unit uniform load. Since there can be any number of lines which inclose the same area, it is possible to have more than one correct influence line. The influence line is for a moving unit load and a particular set of support reactions. If the location of the pin and hinge supports on the model is changed, there will be a corresponding change in the shape of the influence line. Figure 14 shows the shape of the influence lines when the model is supported as indicated. Figure 15 shows the shape of corresponding influence lines for a different set of model supports. If the vertical ordinates of the influence lines are multiplied by the external loads shown on Figures 12 and 13, lines are obtained as shown on Figures 16 and 17. The sum of the areas shown on Figure 16 equals the sum of the areas shown on Figure 17, which confirms that we obtain the same final answer for a given loading regardless of where the model is supported. It was stated earlier that influence lines may be used to illustrate the effect of loads and their location on the structure. This statement is true but it should be applied with caution. It is impossible to add a load to the structure without applying equal loads or reactions elsewhere to keep the structure in equilibrium. When the influence line is used

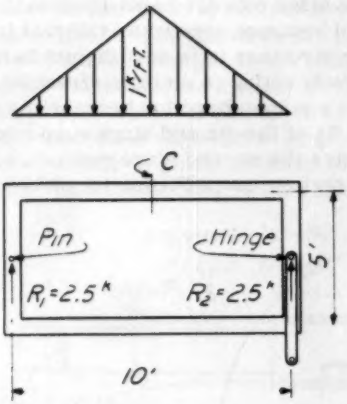


Figure 12

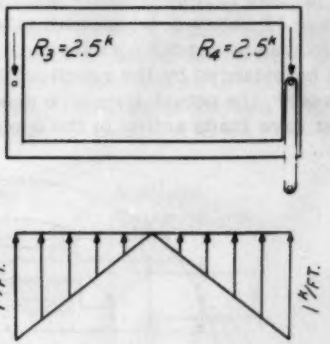
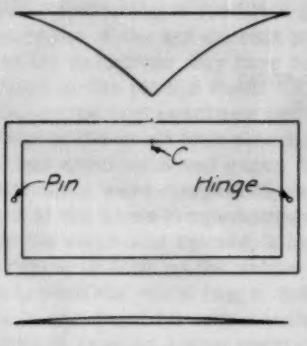


Figure 13

to illustrate the effect of a load, the effect of the balancing load must also be determined and considered.

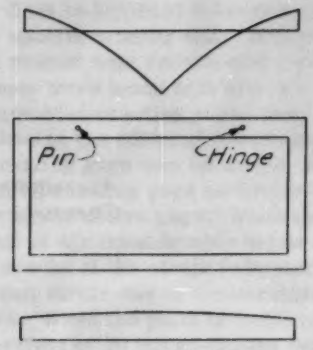
If there is an axis of symmetry, the supports may be located along this line and only one side of the structure need be analyzed experimentally.

The model may be loaded by any means which is convenient. A method which has been found to be satisfactory is to mount the model vertically and to hang a weight from successive points along a loaded surface. The weight is supported by a small pin which passes through a hole near the loaded edge of the model. Holes from which the weight may be hung are drilled at intervals as required to develop the shape of the influence line. It is essential that



Influence Lines for Mom. at C.

Figure 14



Influence Lines for Mom. at C.

Figure 15

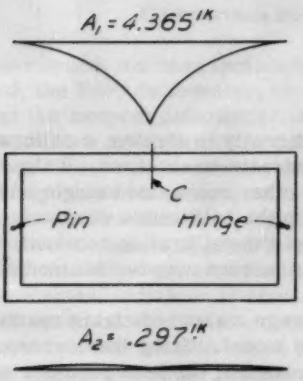


Figure 16

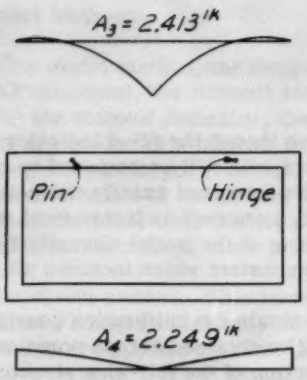


Figure 17

the load acts in the plane of the model. The weight is so proportioned that its center of gravity lies in the plane of the model. The load shown in Figure 18 weighs about five pounds. The exact weight is not critical. The weight should be heavy enough to give a good reading on the strain gage, but not so heavy as to cause buckling of the model. The loading pin is a piece of high strength steel obtained from a number 53 twist drill. The holes along the edge of the model are slightly larger than the pin on the loading weight. Both the holes and the pin must be oriented perpendicular to the plane of the model.[3]

3. Equipment is being developed which will measure the ratio of the strain in the model to the pressure on a hand-held probe. The probe will replace the weights now being used. The model may then be mounted horizontally and floated on ball bearings above a sheet of plate glass.

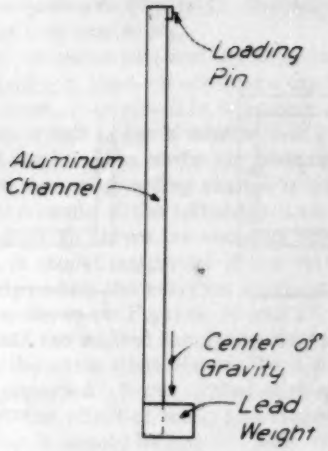


Figure 18

Calibration

Even though the SR-4 indicator may read directly in strains, a calibration constant will still be required to convert the strains to stresses. If the applied load was not exactly one pound or some other recognized single unit of weight, a correction factor must be included in the calibration constant. The thickness of the model also affects the value of the calibration constant. A single constant which includes all of the above factors may be obtained as follows:

To obtain the calibration constant, the average measured strain readings should be obtained across some section of the model. Using the corresponding section of the full-size structure, it is known that the load across a section must equal the algebraic sum of all the loads on either side of the section. This is essential to satisfy the laws of static equilibrium. Knowing the area of the section, and assuming a uniform unit loading, the average stress may be determined for the condition of uniform unit loading. The relationship between the average strain reading on the model and the average stress reading of the full-size structure establishes the value of the calibration constant.

It is well to check the calibration constant by applying statics to several different sections. In the event that the calibration constant does not check, the constant may also be obtained by mounting an SR-4 gage on a simple tension specimen of the same plastic as the model, and then applying known loads.

Effect of Ambient Conditions

While load tests were being made on the various types of plastic, observations were also made on the effect of ambient conditions on measured strain. All load tests were made in an air-conditioned room. The only space available was in an area where there was a noticeable current of air. It was observed that when the operator stood near the SR-4 gage, there was a

noticeable deflection of the SR-4 indicator. This is believed to be caused by a disturbance of the air current passing the specimen under test. It is possible that the deflection may have been due to radiant heat received by the active SR-4 gage or the plastic itself. All of the tests were made with active gages mounted on the test specimen and temperature compensating gages mounted on blanks of the same type plastic. To emphasize the effect of temperature on the test specimens and gages, the compensating gage was insulated, after all other tests were completed, so that the compensating gage no longer was exposed to the same temperature conditions as the active gage. When the door of the room was opened, allowing a draft of air considerably below room temperature to blow on the active gage, the needle of the strain indicator moved beyond the scale range, indicating a unit strain due to temperature change in excess of 50 micro-inches per inch. When the palm of the hand was brought near an active specimen, a deflection of 10 micro-inches per inch was observed. It may be concluded from the above observations that both the active gage and the compensating gage must be at exactly the same temperature and protected from air currents and sources of radiant heat such as direct sunlight.

Comparison with Other Model Methods

Other model methods include the brass-wire model method, the Beggs method, the Eney deformeter, the Gottschalk Continostat, the moment indicator and the moment deformeter.(2) Except for the moment indicator, the above methods are indirect in that the models are loaded in a manner which bears no direct relation to the actual loading on the prototype. They are different experimental techniques of utilizing Maxwell's law of reciprocal deflections. Rather than applying loads directly to the structural model, a known deformation is applied to the model and deformations at other points are determined by measurements.

The moment indicator is considered to be a direct method. This instrument is based on equations applicable to members which are initially straight and have constant I and to which no external loads are applied between the ends of the portion under consideration.

The plastic model with SR-4 strain gages is loaded in a manner which is simple and direct. Loads may be applied either externally or internally and at any point in the frame. The strains may be measured at haunches or other points of stress concentration as well as at the mid-spans. The measured strains are determined at the extreme fibers rather than for some less critical internal area.

With some model methods as, for example, the Beggs deformeter, the frame must be cut at the point to be analyzed. Since it is difficult to put material back which has been cut out, a new model may be required for every point analyzed. The SR-4 equipment can analyze any number of points on the same model. The indirect methods usually employ clamps to deform the model. These clamps immobilize the area which they cover and change the elastic properties of the model. The SR-4 equipment does not interfere with the normal structural action. The choice of the proper sign tends to be confusing and a source of error with most model methods. The SR-4 indicator reads directly in increasing or decreasing strain for an applied load and the nature of the strain is immediately apparent. The SR-4 indicator gives a rapid, easily read, reading. Micrometer microscopes are used in some of

the other methods to measure deflection. Readings taken through a microscope take more time and tend to tire the observer more than similar readings on the SR-4 indicator.

Use on Actual Problem

At Oahe Dam, on the Missouri River, a vertical control shaft is to be constructed for each of the six outlet works tunnels. These shafts vary in depth from two hundred five feet to two hundred thirty-five feet below the ground surface. The shafts have an outside diameter of thirty-eight feet. The shaft structure is divided by cross walls to provide a well for the emergency control gate, a well for the service control gate, and an air vent. A cross section of the shaft structure is shown on Figure 19.

The shaft structure is indeterminate to the sixth degree. The various types of load acting on the structure can be combined into many combinations of loading, any one of which may be severe. These factors, coupled with the massive proportions of the structure, made an exact analytical analysis of the shaft very time-consuming and led to the use of various convenient simplifications in analyzing the structure. It was immediately recognized that some other method of analyzing the structure was desirable.

A plastic model of the shaft cross section was cut from one-eighth inch Plexiglas to a scale of a half inch equals one foot. No simplifying assumptions were made relating to the geometry of deformations. The structure was analyzed for six different nonuniform external loads, two uniform external loads, internal load due to the emergency gate well being flooded to reservoir pool elevation and internal load due to both gate wells flooded to reservoir pool elevation. The effect of pore pressure in the concrete, assuming a uniform hydrostatic gradient between the surfaces of all sections was also considered as a load. All loads were considered individually, first assuming that block-out concrete around the gate track and guide acted monolithically with the structure. The loads were then considered assuming that block-out concrete would not work with the structure. After the structure had been analyzed for the different individual loads, the results were combined to obtain the most severe conditions.

Strain gages were placed at eight points on the structural frame. All the experimental readings required to develop the influence lines for strain at a single gage; and considering both internal and external loads, were normally taken by two men in about one and one-half hours.

Where loads were placed over the entire frame, the experimental method could be checked fairly close by assuming a cut section and equating all the loads on one side of the section to a summation of the stresses. However, it was possible to place localized loads on the structure in such manner that the experimental solution would not check by statics. To insure that the equipment was operating properly, all experimental work was repeated three or more times, some of the strain gages were replaced, and several different support systems were tried. In all cases the measured strains for a given loading checked previous values without difficulty, although a good check by statics was still not obtained. The model was finally cut into sections which were statically determinate and each gage was given an individual check. From this last operation it was concluded that all gages were operating correctly.

In making the check by statics, it was first assumed that there was a linear

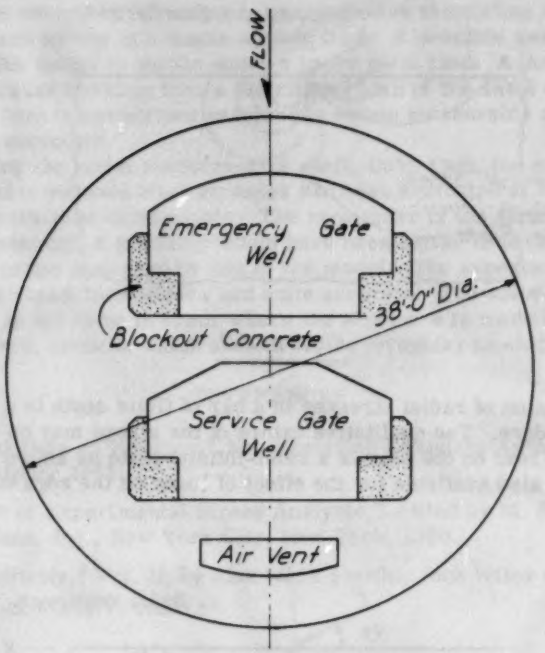


Figure 19

distribution of stress between the extreme fibers. This assumption was in error. It is known that stress distribution is not linear for curved beams. The Oahe shaft structure contains several curved members where the extreme fiber stresses may be expected to differ by about 7 percent from a linear distribution. Haunches, sudden variations in depth of the beam and the shape of the connecting members at the end of each individual beam also cause nonlinear distribution of stress, although the effect of these latter disturbances tends to be localized in the area surrounding the point of origin of the disturbance and not farther from the point of origin than the depth of the beam.

It was observed that high strains were measured when loads were applied close to and in line with a SR-4 gage. An analytical solution to be correct should show comparable strains close to a loaded point. It is known that a radial set of stresses emanate from a point of load application as shown on Figure 20. These radial stresses are neglected in the elementary beam theory ordinarily used in obtaining an analytical solution. At distances from the load greater than the depth of the beam, the effect of the radial stresses is of little importance. But the frame members of the Oahe shaft structure are short and heavy and much of the load is applied close to the point to be analyzed. If the only loads considered are those which are close to the point analyzed, the analytical solution cannot be expected to check the experimental solution unless the radial stresses are determined and superimposed upon the stresses obtained from the elementary beam theory.

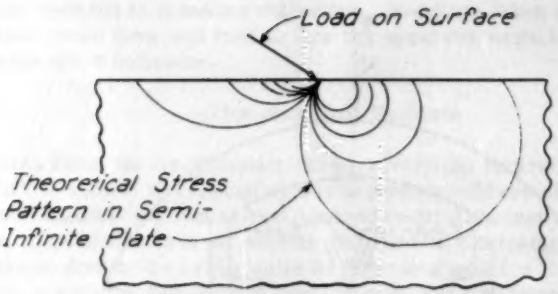


Figure 20

The calculation of radial stresses in a bar of finite depth is a long and difficult procedure. The qualitative nature of the stress may be visualized by assuming a load on the edge of a semi-infinite plate as shown on Figure 21. Equations are also available for the effect of loads on the edge of a circular disk.(3)

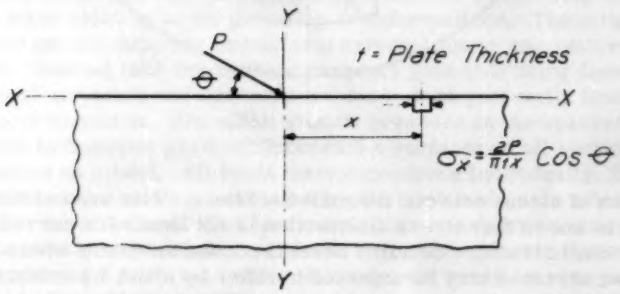


Figure 21

It was concluded that the extreme fiber stresses found by the experimental method are accurate within 5 or 10 percent, assuming a homogeneous elastic structure. While reinforced concrete does not always act as a homogeneous elastic material, the model has contributed a great deal to an understanding of the structural behavior. As before stated, the structure was first designed by analytical methods. As a result of the model analysis, several areas were discovered which might have been critically stressed under certain conditions of loading. The location of some of the reinforcing steel was changed and the total quantity of reinforcing steel used in the shafts was reduced about 2 percent.

CONCLUSION

A satisfactory structural analysis was obtained by model test, employing a two-dimensional model and resistance-type strain gages. A valuable

feature of the described technique is the method of simulating a complex system of loads by use of a single moving load. A possible weakness is the tendency of the model to buckle under a heavy point load. A point load is more apt to cause buckling than a distributed load of the same total value. A fairly heavy load is sometimes required to obtain measurable strain at all points of the structure.

In analyzing the outlet works control shaft, Oahe Dam, the experimental method was discontinued after stresses had been evaluated at enough points to make the structure determinate. The remainder of the structure was then analyzed by statics. It probably would have been better to have analyzed more points in the structure by use of the model. The experimental method of solution is rapid, inexpensive and quite accurate. The solution obtained from statics is apt to be in error where the structure is made up of short heavy members, some of which are curved or irregular in shape.

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JOURNAL

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

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Discussion of "CONTINUOUS PRESTRESSING"

by Robert B. B. Moorman
(Proc. Paper 588)

ROBERT B. B. MOORMAN,[1] M. ASCE.—The writer wishes to thank Messrs. Diwan and Mensch for their contributions by way of discussions of his paper.

Contrary to Mr. Diwan's statements the equivalent load is not of limited value and the eccentricity does not have to be a second degree parabola. However, if the eccentricity is a second degree parabola the equivalent load is uniform.

For any variation of the eccentricity the equivalent load may be computed by use of the expression

$$w_p = T\left(\frac{e_1 - 2e_2 + e_3}{(\Delta x)^2}\right)$$

The foregoing expression gives the intensity of w_p along the member, whether it be variable or uniform.

Furthermore it is not necessary to have coefficients for fixed-end moments for partially loaded beams available. The fixed-end moments, for beams of constant section, may be computed by use of

$$M_{FA} = -\sum \frac{Pab^2}{L^2} = -\sum \frac{(w_p \Delta x)ab^2}{L^2}$$

and

$$M_{FB} = -\sum \frac{Pa^2b}{L^2} = -\sum \frac{(w_p \Delta x)a^2b}{L^2}$$

The dimensions a, b, and L are shown in Fig. 1

The reader should not be misled by Mr. Diwan's Fig. 3, for the final bending moment diagram represents only the effects of the load p and the change in tension in the tendon.

It should be noted that the equivalent load method aids in the analysis for shearing forces as well as bending moments. Also, any method of analysis, classical or modern, may be used when equivalent loads are used.

1. Prof. and Chairman, Dept. of Civ. Eng., Syracuse Univ., Syracuse, N. Y.

Mr. Mensch presents an interesting but brief discussion of ultimate moment. The author believes that safety, unless carried to extreme, is not a fault.

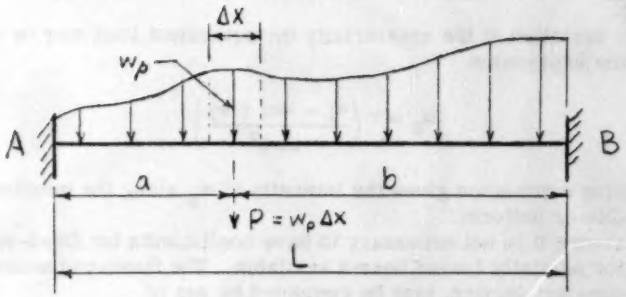

Fig. 1

Discussion of "INFLUENCE LINES FOR MOMENT AND SHEAR IN A CONTINUOUS BEAM"

by Anthony Hoadley
(Proc. Paper 734)

J. J. POLIVKA,[a] M. ASCE.—The paper is a valuable contribution to determining influence lines for continuous beams and frame structures which are of importance for safer analysis of long-span structures under concentrated live load. In such cases the influence lines for several sections in each span are to be determined (usually three or five equidistant sections in each span), so that a greater number of basic calculations is necessary. The author is using the method of moment distribution. For the complete analysis of the author's example (four-span continuous beam, Fig. 1A) at least 12 distribution tables would be required, a time consuming operation, especially if recently proposed simplifications of the distribution method are not applied.[1] Besides, the distribution methods have the disadvantage that the checking of the accuracy is possible after completed operation, and, if the checked values are not correct, the whole distribution operation must be repeated. The determination of influence lines is a specific case where the moment distribution method (as auxiliary step of the problem) should not be used. The writer presented a method which determines basic values valid for any section in any span at the very beginning of the analysis so that many repetitions of calculations are eliminated. This method permits also checking of the accuracy at any important step of the analytical operation and has the advantage of absolute correctness of the results if required (especially for reliable checking).[2]

a. Cons. Engr., Berkeley, Calif.; Lecturer, Stanford Univ., Stanford, Calif.

1. Adrian Pauw, A.M. ASCE: Sequence Summation Factors, Paper No. 763 and references Proc. ASCE, August, 1955;
Kani: Analysis of Multiple-story framework (Die Berechnung mehrstöckiger Rahmen), Stuttgart, 1949, Wittwer;
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K. Heinz Herber: Simplified Kani's Method of Frame Analysis and Improvement of the Convergence, Die Bautechnik, Sept. 1955, p. 310.

2. J. J. Polivka: Graphical Methods of Analyzing Statically Indeterminate Structures, mimiogr. lectures, University of California, 1940, 2nd ed., 1941, both exhausted. New edition in book form "Simplified Methods of Analyzing Statically Indeterminate Structures" is prepared for printing; the writer's method has been discussed in various papers published in Transa., ASCE, as papers No. 2130 ('42), 2152 ('42), 2224 ('44), 2249 ('45), 2260 ('45), 2262 ('45), 2268 ('46), 2408 ('50).
J. J. Polivka: Continuous Arches on Elastic Supports, Der Brückenbau, '20, vol. 4,5 and 6; Determination of Fixed Points Relating to Elastic Supports of Continuous Struct., Bulletin Techn. de la Suisse Romande, '16, p. 245.

The basic principles of the writer's method are presented on the author's example. The elastic properties of this continuous 4-span beam are expressed by fixed (conjugate) points. For a single fixed span with constant moment of inertia these conjugate points are the third-points, determining the principal axis of the ellipse of elasticity of the beam: $i^2 = L^2/12$, of which the center is the midpoint ($s' = s'' = L/2$, Fig. 9). At this point, 0, the elastic weight of the beam (center of elastic gravity) $G = L/IE$, or simply the relative elastic gravity $G = n \cdot L/I = 1/K$ is supposed to be concentrated. In general, the semi-axis of the ellipse of elasticity is derived from the relationships: $i^2 = s'(s''-b) = s''(s'-a)$, a and b being the distances of the conjugate points from the left and the right support respectively. For a fully restrained (fixed) beam $s' = s'' = L/2$, and $a = b = L/3$. The individual spans of a continuous beam or of any member of a framed structure are partially restrained at interior supports, so that the conjugate points are no longer the third points of the span, however the same relations exist. The elastic yielding is expressed by angles of rotation at the support, θ. In the author's example, the relative elastic gravities of the individual (separated) beams are, starting from the restrained support A, 1, 1, 5/6, 2/3. Since the support A is fixed, $a_1 = L_1/3 = 20$ ft., the angle of elastic rotation at B is $B = G_1/4 = 1/4$, and $a_1 = 60{:}3 = 20$ ft. It is easy to prove that $a_2 = L/3 \dfrac{C_2}{G_2 + 2}\theta_B = 200/9$, and that the elastic rotation at D is $\theta_D = G_2/6.\left(3 - \dfrac{L_2}{L_2} - a_2\right) = 2/7$. In similar way a_3 and a_4 are determined. Proceeding from the right free support H, $\theta_G = G_4/3 = 2/3 \times 1/3 = 2/9$ and $b_3 = 50/3 \dfrac{5/6}{5/6 + 4/9} = 250/23$. It is evident that the final moments can be determined directly and the long procedure of distributing fix-end moments is eliminated. The reduction factors are expressed by ratios a:(L - a) and b:(L - b). So e.g. the reduction factor $r_{BA} = 1/2$, $r_{EF} = 5/18$ etc. The final moments in the second span CD produced by unity rotation at the point P for which the influence lines are sought are directly found as follows. The relative gravity of this elastically restrained span, $G_2' = G_2 + \theta_C + \theta_D = 1.489$ is supposed to be concentrated at the center of the ellipse of elasticity 0_2, which is at a distance from the support C: $s_2' = 49.637'$, and from the support D: $s_2'' = 100 - s_2' = 50.363'$. The distance of the point P from the center 0_2 is 9.637. The angular rotation $\theta = 1$ at P is produced by an imaginary force P_2 acting along the antipolar of the point P with respect to the EE (ellipse of elasticity)of the elastically restrained beam CD. The simplest way is the graphical method, as shown in Fig. 10. Two semi-circles above the diameters CY_2'' and $Y_2'D$ intersect at the point $0_2'$ which determines both the centroid 0_2 which EE and its semi-axis $0_20_2' = i_2$. The line $0_2' D_2''$ under the right angle to the line $Y_2'0_2'$ intersects the axis of the continuous beam at the point which locates the line of the imaginary force P_2. From the relation $\theta_2 = P_2 \cdot G_2' \cdot d_2'' = 1$ (unity rotation) the intensity of P_2 is determined: $P_2 = 1{:}(G_2' \cdot d_2'')$. Since, as previously explained, $i_2^2 = 50.363 \cdot 27.415 = 1380.698$, and hence $d_2'' = 1380.698 : 9.637 = 143.266$, the resulting intensity of P_2 is $P_2 = 55.8 : 11{,}905 = 0.0046871$ or, multiplying by 1000 as the author did, $P = 4.687$, and the moment at D, $M_D = P_2 \cdot (d_2'' - s_2'') = 435.447$. Similarly, the moment at C is $M_C = P_2 \cdot (d_2'' + s_2') = 904.157$ or, for checking, $M_C = M_D + 100\ P_2 = 435.447 + 468.710$. These values confirm the results indicated in the paper. Slight differences are due to use of slide rule by the

author. Another check can be made by using the aforementioned principles for the fixed beam CD, from which the author begins the basic calculation. In this case the conjugate points are the third-points of the beam, 0_2 is the mid-point of the span CD, $i_2^2 = L_2^2/12$, the eccentricity of the section P is 50 - 40 = 10, the imaginary force $P_2 = 1000:250/3 = 12$, and the moments $M_D = 12.100/3 = 400$, and $M_C = 1600$, as calculated by the author. The further procedure in the analysis remains the same. It is only suggested, for simplification, to determine the ordinates of the moment influence lines by resolving the imaginary loads in rectangles and triangles, the corner of the latter being always at the left support of each span, since the pertaining moment formulas to be found in every handbook are presented under this condition. These formulas are: (1) for rectangular imaginary load $y_r = \frac{Mx}{2}(L - x)$, (2) for triangular imaginary load $y_t = \frac{Mx}{6L}(L^2 - x^2)$. So e.g. the equation of the moment influence line in the first span can be expressed by the simple formula $y_1 = \frac{M}{2} A \frac{x^2}{L}(L - x) = (-750.345/120)\, x^2 (L - x) = -375.1725\, x^2 + 6.252875\, x^3$ or $y_1 = -13.506 \left(\frac{x}{60}\right)^2 + 13.506 \left(\frac{x}{60}\right)^3$ which agrees with the author's calculations. - This method offers similar simplifications for determining shear influence lines, as it is discussed in the writer's book. The effect of supporting columns rigidly connected with the continuous beam depends on their stiffness and the degree of restraint at the base. In the author's example the stiffness of the columns is very low compared with the stiffness of the beam in its different spans and therefore the effect of the columns on ordinates of the influence lines is relatively small. The same simplifications (as discussed before) can be used, not only for beams on elastic columns but also for more complicated and multi-story frames by determining two conjugate points for each individual structural member. In such cases the graphical application of the writer's method simplifies and shortens the analysis considerably.[3]

3. J. J. Polivka: "Analysis of Complicated Frame structures by Conjugate Lines of Adjacent Members, publ. by Technical University, Prague, 1934; and "Moment Distribution," Engineering News-Record, Aug. 9, 1945.

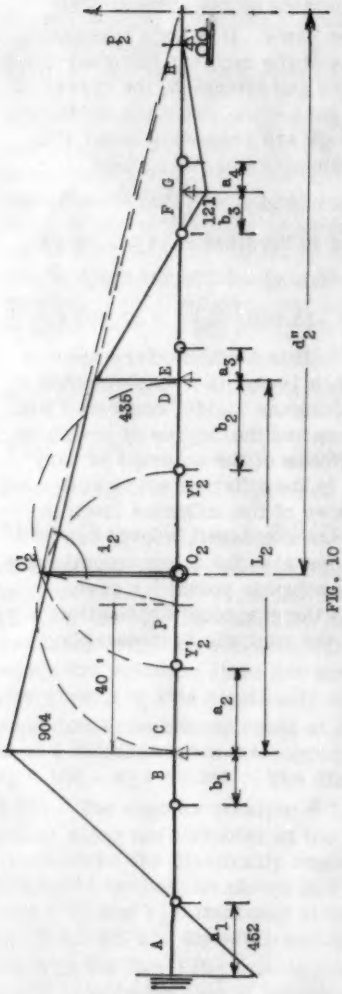

P2
H
G
F
121
b3
a4
d"2
E
D
436
a3
b2
Y"2
i2
O'2
O2
L2
P
Y'2
40'
904
C
a2
B
b1
A
a1
462

FIG. 10

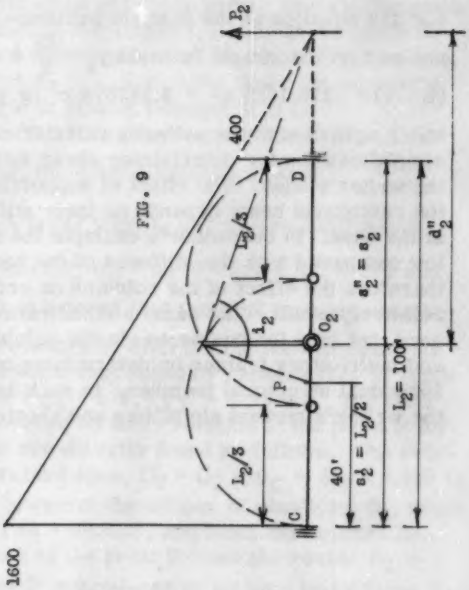

P2
400
D
L2/3
i2
O2
a"2 = a'2
d"2
P
L2/3
40'
a'2 = L2/2
L2= 100'
C
1600

FIG. 9

Discussion of
"INVESTIGATION OF FLOORBEAM HANGERS
IN RAILROAD TRUSSES"

by C. H. Sandberg
(Proc. Paper 762)

J. J. POLIVKA,[1] M. ASCE.—The survey and the analytical and experimental investigations of reported failures are of great interest and importance to all bridge engineers and builders. The writer believes that especially two causes should be taken in consideration in design, fabrication, erection and maintenance of steel bridges: (1) Fatigue of material. Steel bridges, compared with concrete bridges, have relatively low dead load so that the dynamic effects causing constant changes of stresses are serious. It is known that considerable stress changes, sometimes of reverse sign, decrease the strength of steel, and that after a great number of cycles the materials lose a great amount of their original strength due to crystallographic changes as can be found by microscopic examination. (2) Stress concentrations. Greater attention should be paid to various concentrations of stresses caused not only by structural arrangement of connections but also by the shape of cuts and penetration. These causes can be investigated most efficiently and reliably by experimental stress analysis, although considerable studies and research have been done in the theoretical field, E.g., it has been proved that, under certain circumstances, sharp re-entrant cuts in structural members subjected to relatively low stresses show critical stress concentrations under which the failure must occur.[2]

1. Cons. Engr., Berkeley, Calif.; Lecturer, Stanford Univ., Stanford, Calif.
2. J. J. Polivka and H. D. Eberhart, Univ. of California: "Some Stress Relationships in Photoelasticity," Proc. of the 10th Semi-Annual Eastern Photoelasticity Conference, 1939;
J. J. Polivka and H. D. Eberhart: "A more Exact Method of Determining Stresses from Photoelastic Isochromatics and Isoclinics," Proc. of the 15th Semi-Annual Eastern Photoelasticity Conference, 1942;
J. J. Polivka: "Use of Photoelasticity in Analysis of Hyperstatic Structures," Proc. of the 13th S.-A. E. Photoelasticity Conference, 1941.

Discussion of "SEQUENCE SUMMATION FACTORS"

by Adrian Pauw
(Proc. Paper 763)

J. J. POLIVKA,[1] M. ASCE.—The paper is an improvement of the moment distribution method, and any simplification of this lengthy, rather complicated and approximate method is to be commended. As the writer mentioned already in several publications,[2] most of the larger structures are to be analyzed for various types of loading, and the moment distribution method is to be carried out for each type of loading from the very beginning. Furthermore, the checking of the accuracy is only possible when the lengthy process of the analysis is terminated. For these reasons methods of analysis should be preferred by which a great part of calculations can be used for any type of loading (as characteristic constants of the particular framework) and the accuracy can be checked at any stage of the analysis. The writer checked the author's example in Figs. 2 and 8 by his own method and found the results very accurate.

1. Cons. Engr., Berkeley, Calif.; Lecturer, Stanford Univ., Stanford, Calif.; formerly Research Associate in Civ. Eng., Univ. of California, Berkeley, Calif.
2. See references in the writer's discussion of Paper No. 679: "Analysis of Continuous Structures by Joint Rotation" by Charles Y.G. Looney, pp. 831-6 to 831-13.

Discussion of "PLASTIC STRENGTH OF STEEL FRAMES"

by Lynn S. Beedle
(Proc. Paper 764)

JACK R. BENJAMIN,[1] A.M. ASCE.—This is an extremely important and timely paper. It is unfortunate that it could not have included more information on joint details and how they should be designed. Much of this information could be applied to advantage with elastic design procedures.

Unfortunately, this paper gives the impression that the profession could safely adopt plastic procedures at the present time. The writer believes that plastic procedures can only be applied to special small structures. The restrictions on plastic methods must be recognized by the proponents of the procedures.

The most important limitation on plastic procedures is that we do not have sufficient knowledge. Tapered sections or members where a considerable length of member is designed to pass through yielding at the same time have not been studied. Tier buildings cannot be handled until typical large column sections have been tested under conditions of high axial stress. Unfortunately, the limited knowledge available on columns was not included in the paper. Another difficulty with building frames is that the mode of failure of the entire structure at a particular floor must be known. Individual bent strengths may not be attained under conditions of overall torsional failure.

Economy must be considered from two standpoints. Possible savings in a structure will depend entirely on the code load factors. An overall factor of 2 will not produce enough saving over present procedures to warrant the adoption of plastic design procedures. A load factor of less than 2 is questionable unless the loads are known and invariable. Furthermore, the economic problems of the design office must be considered. Plastic design procedures are advocated as faster and better than elastic procedures. Plastic designs will be better balanced with respect to factor of safety but the office design costs will rise rather than decrease. Engineers will have to be re-educated. This takes time and costs money. Furthermore, the procedures of plastic analysis given assume one set of loads that increase gradually up to incipient collapse. In practice we deal with three classes of loads. Dead loads are fixed and are subject only to very minor variation. Vertical live loads are generally known with a certain probability of change. Unknown loads, wind, quake, and blast tend to be in the nature of allowances to insure a certain type of construction. How are these different quantities to be combined in an overall design? The load factor should vary with the type of load under consideration. A complete re-examination of code provisions is necessary. Certainly, the 33 per cent overload provision with wind and

1. Associate Prof. of Civ. Eng., Stanford Univ., Stanford, Calif.

quake should be discarded in favor of a more realistic loading pattern. Plastic design procedures offer the only way to affect a major improvement in the design of steel structure. Adoption of these procedures should include a complete revision of codes if full advantage is to be obtained.

Discussion of "TIGHTENING HIGH-STRENGTH BOLTS"

by F. P. Drew
(Proc. Paper 786)

ELMO G. PETERSON,[1] J.M. ASCE.—The tensioning of high-strength bolts is a problem that has plagued the aircraft industry since their use began. Many of the bolts are used as tension members to join wings to fuselage, to join outer wings to the center wing and in numerous other locations. The useful life of these bolts is determined by the fatigue life and extensive measures and much research has gone into attempting to extend this useful life. The simplest way is to reduce the number of cycles of stress to one by tightening the bolt to a stress higher than that which it will experience in use. The methods investigated by the author were the only ones being used until recently.

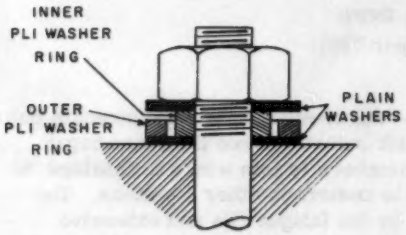
A new method of insuring the proper bolt tension has been developed and is being used by the aircraft industry. It is the use of PLI (preload indicating) washers under the nut or head of the bolt. This preload indicating washer consists of a pair of loosely fitting concentric rings. The inner ring is thicker than the outer ring as shown in Fig. 1. In use the tightening of the nut causes first elastic and then plastic deformation of the inner ring until it is compressed to the same thickness as the outer PLI washer ring as in Fig. 2. This is detected by the use of a feeler handle (a short length of thin gage wire) inserted in one of three holes drilled in the outer circumference of the outer PLI washer ring. The operator wiggles the outer ring during tightening of the nut and stops when the outer ring will no longer move. The compression of the inner ring during the plastic stage occurs with very little increase in the applied load. Hence, the end point is very positive and leaves little or no room for error in human judgment.

The advantages of these washers in any specific use must be weighed against their cost of about sixty to ninety cents each. Any hand or air or electric operated wrench can be used for tightening the bolts. Inspection of the tightness of the joint is also simple and does not require the loosening and retightening with a torque wrench of several of the bolts. The writer feels that they would be ideal for railroad maintenance or any field work where it would be inconvenient to use torque wrenches or calibrated impact wrenches. The savings in time and equipment for calibrating impact wrenches might outweigh their cost on many field construction jobs also.

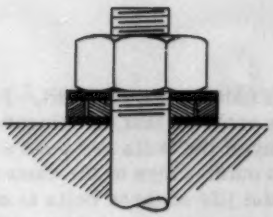
The PLI washers are made of heat-treated, alloy steel. The accuracy of preload is plus or minus ten percent with PLI washers as compared to an average of plus or minus thirty percent with other methods. Hence, the engineer can design and safely figure on at least an additional twenty percent increase in prestress. Reuse of the washers is not recommended since plastic deformation has taken place and the deformation upon reuse will be almost all in the elastic range.

1. Engr., Moffatt and Nichol, Inc., Long Beach, Calif.

Washers for bolts ranging from No. 10 to 1-1/2 inch in diameter and for preloads of 64,000 psi to 192,000 psi are available from the Standard Pressed Steel Company, Jenkintown, Pa., to whom the writer is indebeted for much of the above information.



BEFORE TIGHTENING BOLT
FIG. 1



AFTER TIGHTENING BOLT
FIG. 2

MANLEY A. ROOSE,[1] A.M. ASCE.—It has been the writer's experience that different people have different ideas as to what a "finger tight" nut is. He would like to suggest that to aid in the understanding of just what "finger tight" is, a more specific definition be spelled out. On page 786-6, Mr. Drew describes the "finger tight" or "hand tight" position as ". . . the amount of tightening which can be obtained by turning the nut by hand." This leaves something to the imagination, experience, and judgment of the reader. Does Mr. Drew mean by hand turned up as tight as possible without the use of any tool? Or by "by hand," does he mean manually with a spud wrench and if so, does he mean as tight as it is possible to turn the nut with the wrench, or just how tight? A definition such as the following would clear up such uncertainties:

> " 'Finger tight' is the amount of tightening which can be obtained on the high-strength bolt to be tightened, when in a joint in which the parts are lined up and brought securely together with fit-up bolts, and when the nut is tightened with a spud wrench to the point where the first noticeable resistance to turning is evident. This is to be considered the reference position from which the application of "one turn" tightening is measured."

Now it may be that the writer's definition above does not agree with Mr. Drew's interpretation of the "finger tight" position—if so, the writer is an illustration of the uncertainty that he believes exists about this.

As a more practical measure of the "finger tightened" position, which would rid the bolting operator of the necessity of using both the spud wrench and the impact wrench, perhaps it could be defined as:

> "The tightening obtained on a high-strength bolt in a properly fitted-up joint, when the nut of the bolt is 'spun' to its first snug seating by use of the impact wrench, that being the reference position from which "one turn" of the nut is measured."

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H. P. BUDGEN,[2] M. ASCE.—The Author is to be congratulated in calling to the attention of Engineers the value of using bolts as structural fasteners, and his paper well describes the research carried out on this subject by the Research Council on Riveted and Bolted Structural Joints of the Engineering Foundation.

The exhaustive researches to determine the ideal tightening of high tensile steel bolts, and the plotted results, are most valuable for the permanent, and temporary use of high strength bolts.

It has somewhat surprised the writer that no general use has been made of high tensile bolts before, and the Author's paper does draw attention to this fact. It has been a general rule in England, for the past 20 years, to use fitted High Tensile Bolts tightened with impact wrenches to grip a group of plates, prior to rivetting. One instance with which the writer was concerned occurred on the Howrah Cantilever Bridge, Calcutta, India, when his Company used High Tensile Bolts, filling about one in four of the drilled and reamed holes, which were tightened with pneumatic impact wrenches. As rivetting proceeded, the bolts were released and rivets substituted. This method ensured consolidation of the plates, and assisted the rivets to exert their maximum "contractional grip." As many of the web plates in the top and lower chords of the Cantilever arms consisted of eight or more plates making up to eleven inches in thickness, an impact wrench method of tightening the bolts was essential. All high tensile steel temporary members which were heavily loaded and were generally struts became an integral part of the framework by the use of High Tensile Bolts tightened by the impact wrenches. In a number of steel framed buildings and crane structures, the impact tightened High Tensile Bolt has been used for some years in Britain. In many cases a heavy spring washer has been used to ensure no slackening. The cost saving in site labour when using High Tensile Bolts as compared with rivetting depends upon conditions of working. Where a building is high and column members consist of heavy flanges requiring substantial cover plates, the ideal connections are secured using rivets up to a height of 30/40 feet, but above this height, and for beam connections, High Tensile bolting becomes more economic than rivets. However, in all cases prior to rivetting the High Tensile Bolt should be used to tighten the joints.

The writer experienced difficulties with the pneumatic impact wrench in India. The 'pathans' or 'Hillsmen' are far the best rivetters in India, probably because of some association they have with firearms, and they call a rivetting hammer, or pneumatic wrench a 'gun,' Unfortunately, when tightening a bolt, to stop further tightening, they would move over the reversing lever, and with the wrench tightening one instant, before it had stopped, put it into reverse, which broke the small intermediate gear. Repairs of this item were costly, and it is certainly necessary that ample provision is made with any pneumatic wrench, that it cannot be put readily into reverse.

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Discussion of "REPORT OF ASCE-ACI JOINT COMMITTEE ON ULTIMATE STRENGTH DESIGN"

(Proc. Paper 809)

JACK R. BENJAMIN,[1] A.M. ASCE.—This report is a valuable contribution to reinforced concrete literature. However, the writer believes the committee is on questionable ground insofar as proposing a design procedure is concerned. Structural engineers must deal with overall guaranteed performance. Ultimate strength is a very poor index of such performance. The complete load-deflection curves and photographs of members of particular designs must be studied if the criteria is performance. The useful limit of a particular design is very often short of the ultimate load because of concrete cracking and disintegration. Furthermore, the load-deflection curves for different designs differ greatly, although the ultimate loads may remain essentially constant. With under-reinforced sections the load-deflection curve is essentially a function of the steel alone. The load deflection curve is of Type I, Fig. 1. If the tension steel is increased until a balanced design is approached, the compression failure of the concrete becomes of prime importance producing a rapid drop off of load past ultimate, as shown in curve Type II, Fig. 1. The differences in load-deflection curves are of paramount importance in a design problem involving performance of the structure.

Another basic fallacy involved in the report is contained in the recommendation that an elastic analysis for moments, thrusts, and shears be combined with a design based on ultimate strength. These two items are compatible only with statically determinate structures. The great majority of structures are not statically determinate and the elastic design quantities may bear no relation to the moments, thrusts, and shears present at the failure of even one section much less as the collapse of the entire structure is approached. Elastic design procedures are at least relatively consistent in this respect even though they are inconsistent with respect to factor of safety.

The writer believes the attention of the committee should be directed at the controlled design of reinforced concrete in much the same manner as plastic design is under discussion with steel structures. Plastic design in steel is associated with the development of predictable yield hinges in the structure. The formuli and discussion in this report are not related to the development of yield hinges. In steel, the yield hinge is a useful part of the structure and close inspection is often necessary to determine its exact location. In contrast, reinforced concrete cracks well before ultimate load and the cracking may be so extensive the member is at the limit of its usefulness although the structure may be some distance from collapse.

The writer believes that the problem can only be solved by concentrating on controlled performance. Reinforced concrete sections can be designed to

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behave in a manner similar to a steel section. The cracking can be controlled. Such yield hinge design is a separate and distinct problem from the committee report. The writer has had considerable success in the design of controlled yield hinges on a model scale. A typical detail is given in Fig. 2. This section has characteristics identically parallel to a yield hinge in steel. The section is pre-cracked by the insert shown in Fig. 2. The tension steel is designed to yield over a considerable length without cracking the concrete because the concrete is not bonded to the steel in this region. The compression area of the beam is specially reinforced to control concrete cracking, carry shear, and prevent disintegration under local joint rotation of as much as 10 degrees. The hinge is fixed in location by the designer and made somewhat weaker than adjacent sections. Small beams and rigid frames employing these joints have been tested by graduate students. Fig. 3 contains one typical test result. Note the basic characteristics identical to a steel frame involving yield hinges. Furthermore, the frame test disclosed that all cracking was localized and controlled at the hinges without disintegration up to a total deflection $\frac{1}{20}$ of the span. The writer is not proposing that these limited tests are more than an indication of what can be done. However, they do suggest one possible orientation of the committee's efforts that may yield a rational method of plastic load design in reinforced concrete. The important point is that the complete performance of such a hinge is controlled including appearance.

The arbitrary load factors given in Equations (I) to (IIb) are open to question. The overall load factor is a desirable simplification but would it not be better to vary the load factor with the type of loading? Thus, the live load factor for a roof should differ from that for a warehouse floor. Furthermore, why should these particular formuli be more valid than others of similar intent? Such quantities must be a function of engineering judgment as applied to particular structures under their own loading conditions. Why throw away the freedom to make intelligent design in the interest of uniformity?

The limitation of $p = 0.9$, in Equation 3 for rectangular beams with tensile reinforcement only may not always be satisfactory. The very undesirable characteristics of compression concrete failure make this figure appear to be somewhat high considering the great variations possible in concrete construction.

The complex formuli, Equations 9, etc., are very undesirable. Reinforced concrete is a statistical material. Even with great care strengths are very difficult to predict in the laboratory under ideal test conditions. The writer has been studying reinforced concrete shear walls both mathematically and experimentally since 1951. Rarely can strengths be predicted closer than 10 to 20 per cent. Furthermore, some characteristics defy rational analysis. Complex formuli give the engineer the impression that the product will have a considerable degree of precision. Couldn't much the same results be obtained with much simplified expressions of limited range? These formuli are only as valid as the data from which the emperical coefficients are derived. The great majority of tests involve specimens much smaller than normally encountered in practice. The ranges of data should be quoted with each individual formula. The writer doubts if the very complex mathematical expressions can be completely justified with the experimental data available.

The design of reinforced concrete for ultimate strength in shear must be made on a highly empirical basis. Furthermore, shear failure is most undesirable. Insofar as design is concerned, the writer suggests attention be directed at complete prevention of shear failure rather than balancing the design with moment failure. Tests have been confined to statically determinate structures. Statically indeterminate structures will have to be tested before the complete story can be even approximated. Results with model continuous beams at Stanford have disclosed unusual behavior in many cases at the continuous support. Cracking very often has been largely a function of steel discontinuities rather than apparent moment and shear at the support.

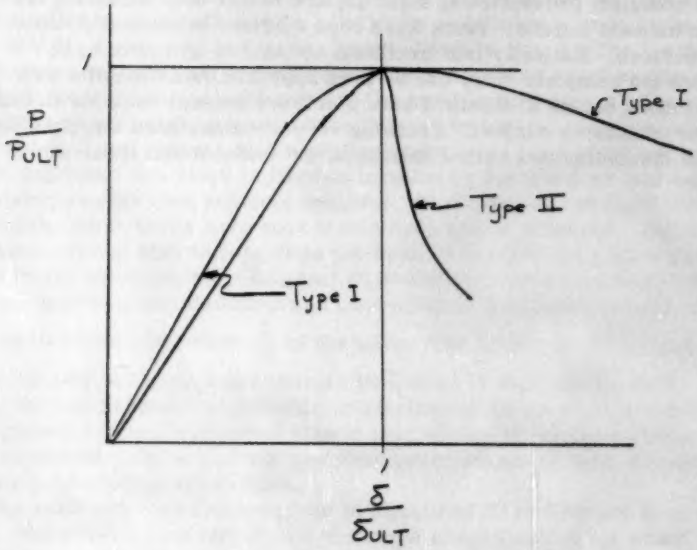

Fig. 1 Load-Deflection Curves

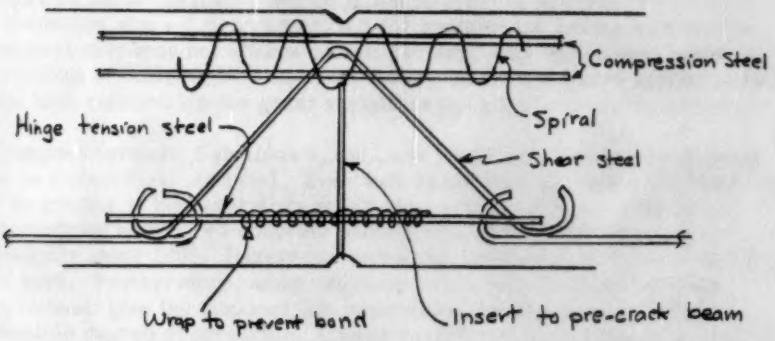

Fig. 2 Yield Hinge Design

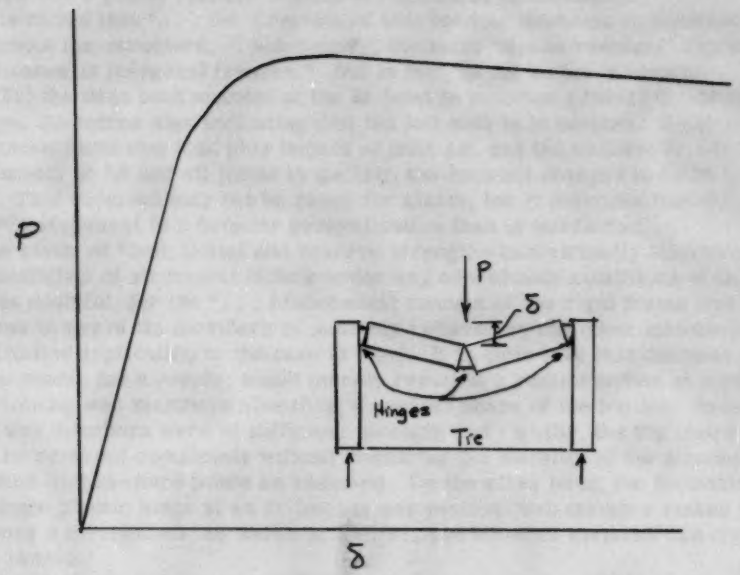

Fig. 3 Load-Deflection Curve of Model Bent

Discussion of "INTRODUCTION TO SEMI-RIGID DETERMINATE POLYGONAL TRUSSES"

by Alexander H. Kenigsberg
(Proc. Paper 828)

GLEN V. BERG,[1] A.M. ASCE.—Replacing the triangular elements of the conventional web system with Y-shaped elements is a novel approach to truss design. That conditions may arise to make such trusses advantageous, either structurally or aesthetically, is not questioned. However, the paper contains a few points that are in need of support or clarification.

It is stated that ". . . the direction of web bending moments is constant throughout the structure. Consequently, the term 'stress reversal' has no significance in polygonal trusses." But in fact, in the author's example (Fig. 5a) the dead load moment at the M-joint in member L10M10 is -3050 In-Kips, the minus sign indicating that the left side is in tension. If one adds the concentrated live load plus impact at joint L8, and the uniform live load plus impact at L8 and all joints to the left, the moment changes to + 526 In-Kips. This reversal may not be cause for alarm, but it indicates that the author's statement is a broader generalization than is warranted.

The claim of "both initial and reserve strengths that virtually eliminate the possibility of structural failure under any conceivable conditions of loading." is doubtful, for the ". . . fundamental concept of the rigid frame that distress in any of its members is instantly relieved by the other members..." is of limited application to the case at hand. It is quite true that distress in the top chord, for example, would merely result in a redistribution of stress, the adjoining web members absorbing a greater share of the burden. Indeed, if the web members were of sufficient strength and rigidity, the top chord could be removed completely without impairing the stability of the structure, even with hinged chord joints as assumed. On the other hand, the formation of a single plastic hinge at an M-joint in any vertical web member makes the structure a mechanism, as shown in Fig. 11, and no other member can come to the rescue.

In the deflection computations by the dummy unit load method, the strain energy due to flexure of web members has been disregarded. Including web flexure, the deflection equation for the author's example (assuming members of uniform section) takes the form

$$D = \sum \frac{SL}{AE} u10 + \sum \frac{ML}{3EI} m10$$

in which m10 is the moment induced in a web member at the M-joint by a unit load at L10, M is the corresponding moment due to dead load, and other terms

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are as used by the author. The last term has been neglected in the paper; yet it appears that its contribution to the deflection is apt to be many times that of the first term as applied to the web system. In view of this, the claims concerning rigidity seem premature. It is hoped that the author will extend the deflection calculations to include the flexural term.

Fig. 11.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. Papers sponsored by the Board of Direction are identified by the symbols (BD). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper numbers are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 861 is identified as 861 (SM1) which indicates that the paper is contained in issue 1 of the Journal of the Soil Mechanics and Foundations Division.

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c. Discussion of several papers, grouped by Divisions.

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